Integrated Decision of Power Transmission Network Planning Based on Interval Vikor Method

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Abstract: Through the establishment of evaluation index system of power transmission network planning, the interval-VIKOR method was applied to power transmission network planning decisions. Based on the principle of the VIKOR method, the group utility and the individual regret calculation formula of program was improved by interval distance formula in the VIKOR method. The principle and steps of the interval-VIKOR method were presented by disposing the practical example of power transmission network planning. Operation is simple, easy to understand, and get a compromise decision scheme to be reasonable, credible, indicating the method has very good value.

Key words: power network planning; VIKOR method; interval numbers

1. Introduction

Multi-attribute decision making is an important aspect of decision analysis theory, solving comparison issue of the merits between the multiple programs with multiple attributes, and it is widely used in engineering, technology, management and other fields. With the development of social economy, the enhancements of the complexity of the objective things and the uncertainty in people's thinking, it is difficult to give the certain evaluation value in the process of evaluation. It can only conduct things of its the pros and cons, good and bad used the approximate values region, usually in the form of interval numbers. Therefore, it has important theoretical and practical significance for this type study of MADA with uncertain interval numbers, and has caused extensive attention of scholars at home and abroad.

Multi-attribute decision making problem exist characteristics, which are the mutual influence between its various attributes and the influence different from each other in solving scheme selection problem with multiple attributes, leading to difficultly find schemes to meet the requirements of all the attributes. So far, many scholars have proposed many solutions to solve the interval numbers MADM problems, frequently used methods are fuzzy comprehensive evaluation method, AHP, gray correlation analysis method, set pair analysis method, projection method, artificial neural network algorithm etc. These methods are either a strong subjective evaluation, or operation is too complex. VIKOR method is a compromise method to solve the MADM problems. on the choice of the program ,it not only take into account the maximization of the group utility and the minimization of the individual regret, but also consider the decision maker's subjective preferences, therefor the decision is more rational. Many scholars have done a significant contribution in this aspect, the literature [1] proposed the fuzzy interval-VIKOR method , the literature [2] proposed extended VIKOR method for dynamic multi-attribute decision making with interval numbers , the literature [3] proposed selection decision of virtual scientific research team member in college and university based on fuzzy VIKOR .

These literatures' interval numbers operation is complicated and error-prone in the process of implementing the interval-VIKOR method. The traditional VIKOR method is extended to the interval numbers in this article, and the group utility and the individual regret calculation formula of program was improved by interval distance formula in the VIKOR method. Operation is simple, easy to understand, and reduces computation. By selecting the transmission network planning schemes demonstrate the effectiveness and feasibility of this method.

2. Interval Numbers' Algorithms

Definition 1: Real axis interval $a=\left[a^L,a^U\right]$, which a^L, a^U are real number, so a is defined as interval number. If $a=\left[a^L,a^U\right]$ = $\left\{x \mid 0<a^L \le x \le a^U\right\}$, so a is defined as positive interval number. Specifically, if $a^L=a^U$, then a degraded to a real number. Under normal circumstances, the interval number is not a real number, so the algorithms of real number cannot be directly used as arithmetic definition of interval numbers, it must be redefined the algorithm of interval number[4].

Definition 2: Let $a=\left[a^L,a^U\right]$, $b=\left[b^L,b^U\right]$ be any two interval number, then interval numbers satisfies the following laws:

Addition: $a+b=\left[a^L+b^L, a^U+b^U\right]$

Subtraction: $a-b=\left[a^L-b^L, a^U-b^U\right]$

Multiplication: $a\cdot b=\left[a^L, a^U\right]\cdot\left[b^L, b^U\right]$
=
$\left[\min\left(a^Lb^L, a^Lb^U, a^Ub^L, a^Ub^U\right), \max\left(a^Lb^L, a^Lb^U, a^Ub^L, a^Ub^U\right)\right]$

Of particular note, when a and b are positive numbers, this formula is changed into: $a\cdot b=\left[a^Lb^L, a^Ub^U\right]$

Division operation:

$$\frac{a}{b}=\frac{\left[a^L, a^U\right]}{\left[b^L, b^U\right]}=\left[a^L, a^U\right]\cdot\left[\frac{1}{b^L}, \frac{1}{b^U}\right]$$

Of particular note, when a and b are positive numbers, this formula is degraded to:

$$\frac{a}{b}=\frac{\left[a^L, a^U\right]}{\left[b^L, b^U\right]}=\left[\frac{a^L}{b^L}, \frac{a^U}{b^U}\right]$$

3. The Basic Idea of the VIKOR Method

The principle of VIKOR algorithm [5,6] is a kind of MADM method proposed by Opricovic in 1998, belongs to an optimal compromise solution method in MADM. The basic idea is first to define the positive ideal solution and negative ideal solution, and then according to the evaluation index to evaluate the closeness between each alternative index value and the ideal scheme index value, then ranking the scheme. VIKOR method obtain a feasible compromised solution which is close to the ideal solution, a compromise implies mutual concessions between multiple attributes. According to the VIKOR algorithm, various sorts of compromise schemes are given by the aggregate function developed by the Lp-metric:

$$\mathrm{L}_{\mathrm{pj}}=\left\{\sum_{\mathrm{i}=1}^{\mathrm{n}}\left[\omega_{\mathrm{i}}\left(f_{\mathrm{i}}^{+}-f_{\mathrm{ij}}\right)/\left(f_{\mathrm{i}}^{+}-f_{\mathrm{i}}^{-}\right)\right]^{\mathrm{p}}\right\}^{1/\mathrm{p}}$$

While, $1\le p\le\infty;\quad i=1,2,\cdots I$.

In the VIKOR algorithm, L_{1i} and $L_{\infty i}$ are used to construct sequential measurements. L_{pi} represents the distance from the program x_i to ideal solution, the biggest characteristic of the VIKOR method is the maximization of the group utility and the minimization of the opposing views individual regret. So that the decision makers are easy to accept the compromise solution. Take evaluation criteria with two attributes for example, VIKOR method's compromise solution can be expressed by the following Figure 1.

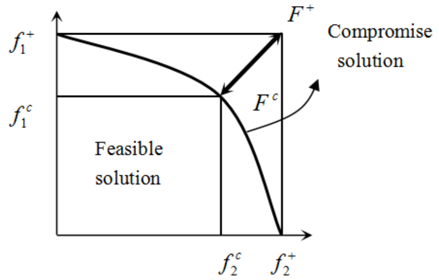

Figure 1. The expression chart of compromise solution.

f_1^+ and f_2^+ respectively represent the ideal solution of the first principle and second principle in the Figure 1. The compromise solution F^c is the most close to the optimal solution F^+ of all solutions, which is the result of mutual compromise between the two principles. And the corresponding compromise is respectively $f_1^+-f_1^c$ and $f_2^+-f_2^c$.

4. The VIKOR Method of Interval Numbers MADM

For MADM problems, the solution of x_i is measured according to the property of u_j, get the attribute value x_{ij} which x_i on u_j (which x_{ij} is interval number, $x_{ij}=\left(x_{ij}^L, x_{ij}^U\right)$), thus constituting decision matrix $X=\left(x_{ij}\right)_{m\times n}$.

$$X=\begin{bmatrix}\left[x_{11}^L, x_{11}^U\right] & \left[x_{12}^L, x_{12}^U\right] & \cdots & \left[x_{1n}^L, x_{1n}^U\right]\\ \left[x_{21}^L, x_{21}^U\right] & \left[x_{22}^L, x_{22}^U\right] & \cdots & \left[x_{2n}^L, x_{2n}^U\right]\\ \cdots & \cdots & \cdots & \cdots\\ \left[x_{m1}^L, x_{m1}^U\right] & \left[x_{m2}^L, x_{m2}^U\right] & \cdots & \left[x_{mn}^L, x_{mn}^U\right]\end{bmatrix}$$

In the matrix, x_{ij} represents indicators value which is the *i*-th alternative of the *j*-th indicators

On the basis of, the traditional VIKOR method thinking, given the calculation steps to solve interval numbers MADM problems which attribute weights are determined.

Step 1 Normalizing the decision matrix. The most common attribute types are benefit type and cost type. Let $I_j\left(j=1,2\right)$ represent benefit type and cost type respectively. In order to eliminate the effects of different physical dimension to the decision result, can use the vector normalization method to transform decision matrix X into matrix $Y=\left(y_{ij}\right)_{m\times n}$, while $y_{ij}=\left(y_{ij}^L, y_{ij}^U\right)$,

$$\begin{cases} y_{ij}^L=x_{ij}^L\Big/\sum_{i=1}^{m}x_{ij}^U\\ y_{ij}^U=x_{ij}^U\Big/\sum_{i=1}^{m}x_{ij}^L \end{cases}\quad i\in m, j\in I_1 \tag{1}$$

$$\begin{cases} y_{ij}^L = \left(1/x_{ij}^U\right) \Big/ \sum_{i=1}^{m}\left(1/x_{ij}^L\right) \\ y_{ij}^U = \left(1/x_{ij}^L\right) \Big/ \sum_{i=1}^{m}\left(1/x_{ij}^U\right) \quad i \in m, j \in I_2 \end{cases} \tag{2}$$

After processing, the cost type indicators have been transformed into benefit type indicators, achieving consistency of indicators attribute.

Step 2 To determine the positive ideal solution f_j^+ and negative ideal solution f_j^- of the indicators,

The positive ideal solution:

$$f_j^+ = \left[f_j^{+L}, f_j^{+U}\right] = \left[\max_i(y_{ij}^L), \max_i(y_{ij}^U)\right], i \in M, j \in N$$

The negative ideal solution:

$$f_j^- = \left[f_j^{-L}, f_j^{-U}\right] = \left[\min_i(y_{ij}^L), \min_i(y_{ij}^U)\right], i \in M, j \in N$$

Step 3 To calculate the group utility $S_i = \left[S_i^L, S_i^U\right]$ and the individual regret $R_i = \left[R_i^L, R_i^U\right]$ of program:

$$S_i = \sum_{j=1}^{n} \omega_j (f_j^+ - y_{ij})/(f_j^+ - f_j^-)$$

$$R_i = \max\left[\omega_j (f_j^+ - y_{ij})/(f_j^+ - f_j^-)\right]$$

While ω_j represents the j-th indicators weight.

$$f_j^+ - y_{ij} = \left|\left[f_j^{+L}, f_j^{+U}\right] - \left[y_{ij}^L, y_{ij}^U\right]\right| = \sqrt{\left(f_j^{+L} - y_{ij}^L\right)^2 + \left(f_j^{+U} - y_{ij}^U\right)^2}$$

$$f_j^+ - f_j^- = \left|\left[f_j^{+L}, f_j^{+U}\right] - \left[f_j^{-L}, f_j^{-U}\right]\right| = \sqrt{\left(f_j^{+L} - f_j^{-L}\right)^2 + \left(f_j^{+U} - f_j^{-U}\right)^2}$$

Step 4 To calculate VIKOR values of program

$$Q_i = v\frac{S_i - S^+}{S^- - S^+} + (1-v)\frac{R_i - R^+}{R^- - R^+}$$

Which,

$$S^+ = \min_i S_i;$$

$$S^- = \max_i S_i;$$

$$R^+ = \min_i R_i;$$

$$R^- = \max_i R_i$$

Q_i represents the *i*-th alternative's VIKOR value. v represents the maximum utility weight, if $v > 0.5$, which means that making decisions according to most people's esolution; when $v = 0.5$, which means that make decisions according to the agreed conditions; when $v < 0.5$, which means that make decisions according to the case of refusal.

Step 5 When meet the following two conditions, solution can be sorted according to the size of Q_i, the solution with the minimum Q_i is final solution.

Condition 1: an acceptable benefit threshold condition

$$Q'' - Q' \geq 1/(m-1)$$

While Q'' is the second solution sorted out by Q, Q' is the first solution sorted out by Q, m represents the number of solutions. This formula indicates the difference between the interest rate between two adjacent solutions must exceed the threshold value $1/(m-1)$, to determine first solution is significantly better than second solution. When there are several options, compared successively whether the solution sorted first, second, third, etc. compliance with the conditions of the 1.

Condition 2: acceptable decision reliability

After sorted according to the value of Q, the value of S ranking in the first must have a good performance than S ranking in the second. Or sorted according to the value of Q, the value of R ranking in the first must be have a good performance than R ranking in the second. When there are several options, compared successively whether the solution sorted first, second, third, etc. compliance with the conditions of the 2.

Evaluation criteria:

If the relationship between solution ranking in the first and ranking in the second meet the conditions both 1 and 2, then the first sort is accepted as the optimal solution. If the relationship between solution rankings in the first and ranking in the second only meet the conditions 2, then both the first sort and the second sort are accepted as the optimal solution. If the relationship between solution ranking in the first and several other solutions do not meet the condition 1, but only qualified condition 2, then these do not meet the condition 1 are accepted as the optimal solutions.

5. Example Analyses

Decision of Power transmission network planning is a multi object decision problem, which is the process of selecting the optimal solutions from feasible solutions based on a number of specific evaluation indicators. Power transmission network planning evaluation index system studies how to establish an effective evaluation system to monitor, guide and manage construction and development of the power network. The established power network planning evaluation index system needs characterization of grid technologies, the economy and the level of development systematically and comprehensively. Therefore, we must deal with the selection and design of the indicators as well as the relationship between indicators and other issues, as far as possible to avoid or reduce duplication of indicators definitions, random choice of indicators, index system is too complex and other problems. Shown in Figure 1 [7,8].

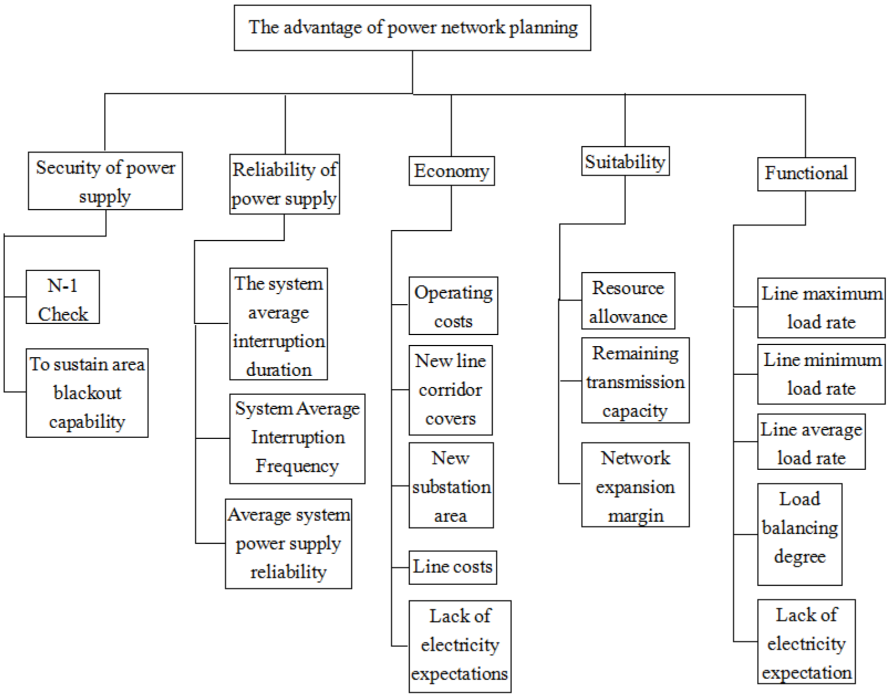

Figure 2. The evaluation index system of transmission network planning.

The IEEE Garver-6 system is often used as a classic example in the field of power transmission network planning at home and abroad. This article analysis based on the case on literature [9]. Assuming each path can stand the line up to three times, each time the line maximum transmission power is 300MW, the cost is (25, 31.25) Million/km, to calculate the corresponding line cost; According to the experimental results of psychologists, the new line corridor covers and new substation area of this kind of qualitative indicators, to identify a number of different objects in an attribute, ordinary people can correctly distinguish attribute level between grade five to grade nine, thus, corresponding to 7 level to describe, respectively, {worst, poor, somewhat less, general, slightly better, better, good}, in order to make the description of alternatives evaluation index of power transmission network planning more accurate , the target property of seven levels is divided into corresponding interval numbers. As shown in Table 1.

Table 1. Level and interval number comparison table.

Grade		Worst	Poor	Somew hat less	General	Slightly better	Better	good
Scores	Lower	2	3	4	5	6	7	8
Scores	Caps	3	4	5	6	7	8	9

Table 2. The index appraising value of alternative scheme of power transmission network planning.

Evaluation	Plan 1	Plan 2	Plan 3
New line corridor covers u_1	[6,7]	[5,6]	[6,7]
New substation area u_2	[6,7]	[3,4]	[3,4]
Construction cost / RMB 10,000 u_3	[5000,6000]	[7450,8000]	[7500,8125]
Operating costs / RMB 10,000 u_4	[700,885.3]	[500,573]	[500,526.7]
Line costs / one million u_5	(50,62.5)	(74.5,93.2)	(67.5,84.4）
Lack of electricity expectations u_6	[2.5,8.8]	（3,6）	（3.5,6.5）
Line maximum load rate/（%） u_7	[80,94]	[86,90]	[70,88]
Line minimum load rate/（%） u_8	[2.3,3.6]	[3.1,6.1]	[2.5,3.4]
Line average load rate/（%） u_9	[65,72.4]	[60.3,70]	[55.4,65]
Remaining transmission capacity/MW u_{10}	[346,512]	[700,832]	[748,850]
N-1check u_{11}	[3,4]	[7,8]	[8,9]
To sustain area blackout capability u_{12}	[2,3]	[6,7]	[5,6]
Resource allowance u_{13}	[8,9]	[3,4]	[5,6]
Network expansion margin u_{14}	[8,9]	[7,8]	[7,8]
Load balancing degree u_{15}	[5,6]	[7,8]	[6,7]
Average system power supply reliability u_{16}	[4,5]	[6,7]	[7,8]
The system average interruption duration u_{17}	[6,7]	[7,8]	[7,8]
System Average Interruption Frequency u_{18}	[5,6]	[6,7]	[5,6]

Now there are three power transmission network planning scheme to accept evaluation x_i（i=1,2,3）, and evaluation is the 18 performance indexes u_j (j = 1,2, ..., 18)as shown above, u_1 - u_6, u_{17}, u_{18} are cost type indicators, the other are benefit type indicators. The weights of 18 indicators respectively are

$$\omega = \{0.00210,0.02600,0.00600,0.01030,0.15490,0.40580,0.15700,0.00230,0.00250,0.06260,$$
$$0.09820,0.04590,0.00094,0.00510,0.01690,0.00140,0.00190,0.00016\}$$

Since then, the power transmission network planning decision problem is a typical interval MADM problem.

Processing on the basis of the above theory:

(1) Normalizing the decision matrix

u_1 - u_6, u_{17}, u_{18} are cost type indicators, the other are benefit type indicators, according to cost type indicators and benefit type indicators respectively, decision matrix X will be processed into decision matrix $Y = (y_{ij})_{m\times n}$,while $y_{ij} = (y_{ij}^L, \ y_{ij}^U)$

$$Y = \begin{bmatrix} (0.26786,0.36842) & (0.17143,0.25926) & (0.35646,0.48223) & (0.20808,0.29928) \\ (0.31250,0.44211) & (0.30000,0.51852) & (0.26734,0.32364) & (0.32148,0.41899) \\ (0.26786,0.36842) & (0.30000,0.51852) & (0.26323,0.32148) & (0.34974,0.41899) \end{bmatrix} \rightarrow$$

$$\begin{matrix} (0.33169,0.51843) & (0.11151,0.92134) & (0.29412,0.39831) & (0.17557,0.45570) \\ (0.22243,0.34794) & (0.16355,0.76779) & (0.31618,0.38136) & (0.23664,0.77215) \\ (0.24562,0.38402) & (0.15097,0.65810) & (0.25735,0.37288) & (0.19084,0.43038) \end{matrix} \rightarrow$$

$$
\begin{matrix}
(0.31340,0.40066) & (0.16101,0.29274) & (0.14286,0.22222) & (0.12500,0.23077) \\
(0.29074,0.38738) & (0.32573,0.47570) & (0.33333,0.44444) & (0.37500,0.53848)\rightarrow \\
(0.26712,0.35971) & (0.34093,0.47380) & (0.38095,0.50000) & (0.31250,0.46154) \\
(0.42105,0.56250) & (0.32000,0.40909) & (0.23810,0.33333) & (0.20000,0.29412) \\
(0.15789,0.25000) & (0.28000,0.36364) & (0.33333,0.44444) & (0.30000,0.41176)\rightarrow \\
(0.26316,0.37500) & (0.28000,0.36364) & (0.28571,0.38889) & (0.35000,0.47059)
\end{matrix}
$$

$$
\left.\begin{matrix}
(0.31579,0.42424) & (0.29412,0.42000) \\
(0.27632,0.36364) & (0.25210,0.35000) \\
(0.27632,0.36364) & (0.29412,0.42000)
\end{matrix}\right]
$$

(2) To determine the positive ideal solution f_j^+ and negative ideal solution f_j^- of the indicators,

The positive ideal solution:

$f_1^+ = (0.31250,0.44211)$, $f_2^+ = (0.30000,0.51852)$, $f_3^+ = (0.35646,0.48223)$, $f_4^+ = (0.34974,0.41899)$, $f_5^+ = (0.33169,0.51843)$, $f_6^+ = (0.16355,0.92134)$, $f_7^+ = (0.31618,0.39861)$, $f_8^+ = (0.23664,0.77215)$, $f_9^+ = (0.31340,0.40066)$, $f_{10}^+ = (0.34093,0.47570)$, $f_{11}^+ = (0.38095,0.50000)$, $f_{12}^+ = (0.37500,0.53848)$, $f_{13}^+ = (0.42105,0.56250)$, $f_{14}^+ = (0.32000,0.40909)$, $f_{15}^+ = (0.33333,0.44444)$, $f_{16}^+ = (0.35000,0.47059)$, $f_{17}^+ = (0.31579,0.42424)$, $f_{18}^+ = (0.29412,0.42000)$

The negative ideal solution:

$f_1^- = (0.26786,0.36842)$, $f_2^- = (0.17143,0.25926)$, $f_3^- = (0.26323,0.32148)$, $f_4^- = (0.20808,0.29928)$, $f_5^- = (0.22243,0.34794)$, $f_6^- = (0.11151,0.65810)$, $f_7^- = (0.25735,0.37288)$, $f_8^- = (0.17557,0.43038)$, $f_9^- = (0.26712,0.35971)$, $f_{10}^- = (0.16101,0.29274)$, $f_{11}^- = (0.14286,0.22222)$, $f_{12}^- = (0.12500,0.23077)$, $f_{13}^- = (0.15789,0.25000)$, $f_{14}^- = (0.28000,0.36364)$, $f_{15}^- = (0.23810,0.33333)$, $f_{16}^- = (0.20000,0.29412)$, $f_{17}^- = (0.27632,0.36364)$, $f_{18}^- = (0.25210,0.35000)$

(3) To calculate the group utility S_i and the individual regret R_i of program

$R_1 = 0.09820$, $R_2 = 0.23222$, $R_3 = 0.39856$;

$S_1 = 0.39108$, $S_2 = 0.47762$, $S_3 = 0.71853$

(4) To calculate VIKOR values of program

Let v = 0.5, $Q_1 = 0$, $Q_2 = 0.35524$, $Q_3 = 1$

(5) Select the final program

The group utility S_i and the individual regret R_i as well as VIKOR values of program are sorted from small to large, the results as shown in the Table 3 below:

Table 3. Evaluation result.

VIKOR values Q_i	Q_1 = 0	Q_2 = 0.35524	Q_3 = 1
the individual regret R_i	R_1 = 0.09820	R_2 = 0.23222	R_3 = 0.39856
The group utility S_i	S_1 = 0.39108	S_2 = 0.47762	S_3 = 0.71853

6. Concluding Remarks

The traditional VIKOR method is extended to the interval numbers in this article, and the group utility and the individual regret calculation formula of program was improved by interval distance formula in the VIKOR method, In the selection of the final decision eliminates the comparison process of interval numbers, the operation is simple, easy to understand, the results intuitively reflect the final result ranking. Through specific examples of transmission network planning scheme, objective and comprehensive power transmission network planning evaluation index system was established, steps is clear, and not only the maximization of the group utility and the minimization of the individual regret are considered when choosing a solution, but also decision maker's subjective preferences are fully reflected, and get a compromise decision scheme to be acceptable, indicating the method has very good application value.

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